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total accumulation

= rate of chg (in accumulation)

TIMES (time)

5.3) The Definite Integral

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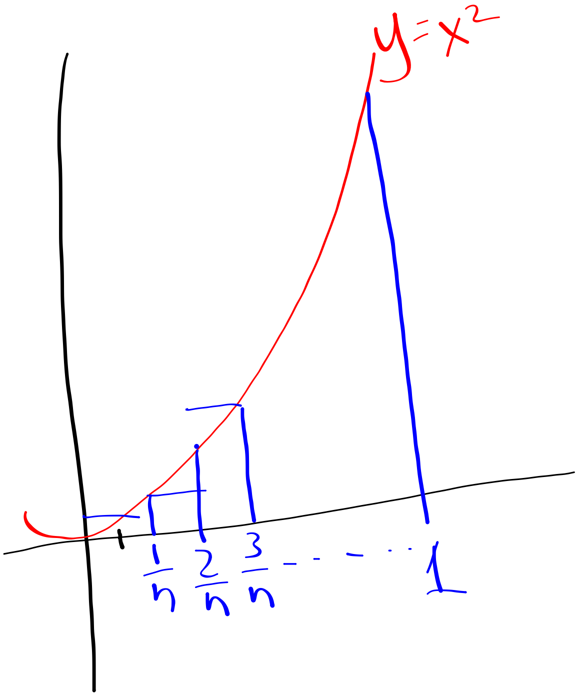

A_{under} =
$y=x^2$
from 0
to 1

$$\frac{1}{n}(h_1+h_2+h_3+\ldots h_n)$$

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The Definite Integral, part 1

the area under a non-negative valued function $f(x)$; above the x-axis, from $x=a$ to $x=b$ is written

$$(\#) \quad \int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n}\left(f(x_k^*)\right)$$

if the limit exists

and "means"

$x_k \to$ an x-value in the k^{th} sub-interval

$* \to$ doesn't matter

do not confuse with

(family of functions) $\int f(x)\,dx$ which means "antiderivative" of $f(x)$

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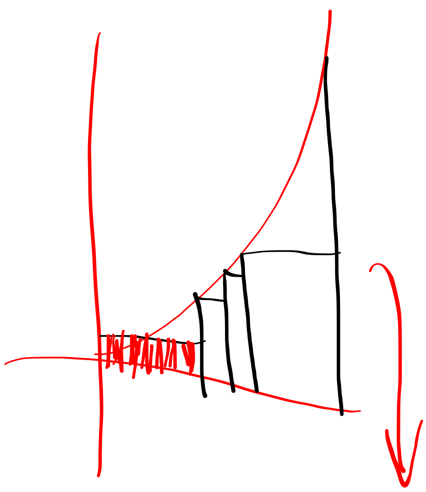
. . . . Part 2.

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} \underbrace{\frac{1}{n}}_{\text{width}} \underbrace{f(x_k^*)}_{\text{height}}$$

changed to

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} \Delta x_k\, f(x_k^*)$$



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The definite integral, part 3 and final

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$$\int_a^b f(x)\,dx = \lim_{\Delta x_k \to 0} \sum_{k=1}^{n} \Delta x_k f(x_k^*)$$

"mesh size $\to 0$"

This general way of dividing an interval into sub-intervals is called partitioning.

A specific example is called a partition

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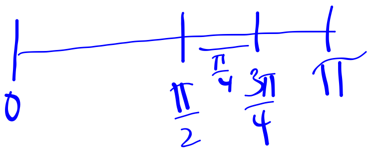
An example (approximation)

Approximate the area under $y = \sin x$ from 0 to π, using a partition of 3 sub-intervals with $\Delta x_1 = \frac{\pi}{2}$



$\Delta x_2 = \Delta x_3 = \frac{\pi}{4}$

Approximate area $= \sum_{k=1}^{3} \Delta x_k f(x_k^*)$

$= \Delta x_1 f(x_1^*) + \Delta x_2 f(x_2^*) + \Delta x_3 f(x_3^*)$

LHS

$= \frac{\pi}{2}(\sin(0)) + \frac{\pi}{4}\left(\sin\left(\frac{\pi}{2}\right)\right) + \frac{\pi}{4}\left(\sin\left(\frac{3\pi}{4}\right)\right)$

$= \frac{\pi}{2}(0) + \frac{\pi}{4}(1) + \frac{\pi}{4}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4} + \frac{\sqrt{2}\pi}{8}$

RHS

$= \frac{\pi}{2}\left(\sin\left(\frac{\pi}{2}\right)\right) + \frac{\pi}{4}\left(\sin\left(\frac{3\pi}{4}\right)\right) + \frac{\pi}{4}(\sin(\pi))$

$= \frac{\pi}{2} + \frac{\pi\sqrt{2}}{8} + 0 \cdots$

MP

$= \frac{\pi}{2}\left(\sin\left(\frac{\pi}{4}\right)\right) + \frac{\pi}{4}\left(\sin\left(\frac{5\pi}{8}\right)\right) + \frac{\pi}{4}\left(\sin\left(\frac{7\pi}{8}\right)\right)$

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4.7/52) $\lim_{x\to\infty} x-\sqrt{x^2+1}$

strategy for $\infty-\infty$

1 fraction

or punt

$= \lim_{x\to\infty} \frac{x-\sqrt{x^2+1}}{1}\cdot\frac{x+\sqrt{x^2+1}}{x+\sqrt{x^2+1}}$

$= \lim_{x\to\infty} \frac{x^2-(x^2+1)}{x+\sqrt{x^2+1}} = \lim_{x\to\infty} \frac{-1}{x+\sqrt{x^2+1}}$

$=0$

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4.7/45) $\lim_{x\to 0} x \csc x$

0 ; $\frac{1}{0}$ = infinity is

$\lim_{x\to 0} \frac{\csc x}{\frac{1}{x}}$

$\stackrel{LH}{=} \lim_{x\to 0} \frac{\csc x \cot x}{\frac{1}{x^2}}$

$\lim_{x\to 0} \frac{x}{\frac{1}{\csc x}} = \lim_{x\to 0} \frac{x}{\sin x}$

$\stackrel{LH}{=} \lim_{x\to 0} \frac{1}{\cos x} = \frac{1}{1}$

$= 1$

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4.7/48

$$\lim_{x\to\infty}\left[\csc\left(\tfrac{1}{x}\right)\right]^{\infty}\left[e^{\frac{1}{x}}-1\right]^{0}$$

$$\lim_{x\to\infty}\frac{\left(e^{\frac{1}{x}}-1\right)}{\frac{1}{\csc\left(\frac{1}{x}\right)}}$$

$$\lim_{x\to\infty}\frac{\csc\left(\frac{1}{x}\right)}{\left(\frac{1}{e^{\frac{1}{x}}-1}\right)}$$

$$=\lim_{x\to\infty}\frac{e^{\frac{1}{x}}-1}{\sin\left(\frac{1}{x}\right)}$$

$$=\lim_{x\to\infty}\frac{e^{\frac{1}{x}}\cdot\frac{d}{dx}\left(\frac{1}{x}\right)}{\cos\left(\frac{1}{x}\right)\cdot\frac{d}{dx}\left(\frac{1}{x}\right)}$$

$$=\lim_{x\to\infty}\frac{e^{\frac{1}{x}}}{\cos\left(\frac{1}{x}\right)}=\frac{e^{0}}{\cos(0)}=\frac{1}{1}=1$$

4.7/55)

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for indeterminate forms

$1^\infty, 0^0, \infty^0$

use ln

or m.p.

$e^{\ln}$

$$\lim_{x\to 0^+} x^{2x}$$

$$=\lim_{x\to 0^+} e^{\ln x^{2x}}$$

$$=\lim_{x\to 0^+} e^{(2x)(\ln x)}$$

$$= e^{\lim_{x\to 0^+}(2x)(\ln x)} \quad e^{\lim_{x\to 0^+}\frac{\ln x}{\frac{1}{2x}}}$$

$$\stackrel{LH}{=} e^{\lim_{x\to 0^+}\frac{\frac{1}{x}}{\frac{-2}{(2x)^2}}}$$

$$= e^{\lim_{x\to 0^+}\frac{\frac{4x^2}{x}}{-2}} = e^{\lim_{x\to 0^+} -2x} = e^0 = 1$$

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